

Do we have the explanation for the Higgs and Yukawa couplings of the *standard model*?

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Abstract

The *spin-charge-family* theory [1–4, 6–8, 25] offers a possible explanation for the assumptions of the *standard model*, interpreting the *standard model* as its low energy effective manifestation [3]. The theory predicts several scalar fields determining masses and mixing matrices of fermions and weak bosons. The scalar fields manifest as doublets with respect to the weak charge, while they are triplets with respect to the family quantum numbers. Since free scalar fields (mass eigen states) differ from those which couple to Z_m and to W_m^\pm or to each family member of each of the family the *spin-charge-family* theory predictions for LHC might differ from those of the *standard model*.

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I. INTRODUCTION

The *standard model* assumes one scalar field, the Higgs (and the anti-Higgs) with the non zero vacuum expectation value and with the weak charge in the fundamental representation. It assumes also the Yukawa couplings. The Higgs (and the anti-Higgs), interacting with the weak and hyper gauge bosons, determines masses of the weak bosons and, "dressing" the right handed fermions with the needed weak and hyper charges, determines, together with the Yukawa couplings, also the masses of the so far observed families of fermions.

The question is: Where do the Higgs together with the Yukawa couplings of the *standard model* originate from? Is the Higgs a scalar field with the fermionic quantum numbers in the charge sector? Or there are several scalar fields (all doublets with respect to the weak charge), which manifest in the observable region as the Higgs and the Yukawa couplings?

To answer any question about the origin of the Higgs and the Yukawa couplings one first needs the answer to the question: Where do families originate? And correspondingly: How many families do we have at all?

There are several inventive proposals in the literature [10–18] extending the *standard model*. No one explains, to my knowledge, the origin of families.

There are several proposals in the literature explaining the mass spectrum and mixing matrices of quarks and leptons [23] and properties of the scalar fields [19–22]. All of them just assuming on one or another way the number of families.

I am proposing the theory [1–4, 6–8, 24] [32], the *spin-charge-family* theory, which does offer the explanation for the origin of families, of vector gauge fields and of several scalar fields: A simple starting action at higher dimensions determines at low energies properties of families of fermions, of vector gauge fields and of several scalar fields. The *standard model* can be interpreted as a low energy manifestation of the *spin-charge-family* theory.

Families appear in the theory due to the fact that there are two kinds of gamma matrices (two kinds of the Clifford algebra objects, only two): i. One kind (γ^a) was used by Dirac to describe the spin of fermions (spinors). ii. The second kind ($\tilde{\gamma}^a$) I am proposing to explain the origin of families [33]. The theory predicts in the low energy regime two decoupled groups of four families. The lowest of the upper four families is the candidate to form the dark matter. Before the electroweak break there are four massless families of (u^i and d^i , $i \in \{1, 2, 3, 4\}$) quarks and (e^i and ν^i) leptons, left handed weak charged and right

handed weak chargeless, which when coupling to massive scalar fields with non zero vacuum expectation values and to gauge fields, become massive.

Vector and scalar fields (both with respect to $(3+1)$), which originate in the spin connections of two kinds (they are gauge fields of the two kinds of the Clifford algebra objects) and vielbeins at higher dimensions, are expected to have charges in the scalar, vector or any adjoint representations with respect to all charge groups, subgroups of the starting group. The question then arises: How can the scalar fields at low energies manifest effectively as weak doublets, as the Higgs certainly does? Although it is not at all simple to show, why the symmetries break in the way that they manifest the observed properties of fermions and vector and scalar bosons, the answer to the question, why scalar fields behave as weak doublets, is simple, if the way of breaking symmetries is assumed.

In the refs. [2, 3, 6] I present the assumed breaks of the starting symmetries of the simple starting action used by the *spin-charge-family* theory, as well as possible answers to the above questions. In this paper I repeat the assumptions of the *spin-charge-family* theory, I briefly represent the low energy manifestation of the simple starting action after the assumed breaks of the starting symmetries, analyse properties of the families: their charges, their coupling to the scalar fields, and to the vector fields, their mass matrices and correspondingly their masses and mixing matrices and masses of the weak bosons when they couple to several scalar fields (following to some extent the refs. [3, 5, 6, 24]), I offer the answer to the question, why do scalar fields appear as doublets with respect to the weak $SU(2)$, while they behave as triplets with respect to the family groups, and I comment on predictions beyond the *standard model*.

Although the *spin-charge-family* theory requires additional studies to be proved (or might be even disproved) that it is the right step beyond the *standard model*, the work done so far [1–4, 7–9, 24, 25] gives a real hope. The theory explains the assumptions of the *standard model*: The spins and charges of quarks and leptons, left and right handed, the families, the Higgs (manifesting as a superposition of several scalar fields), the Yukawa couplings, predicting the fourth family and the stable fifth family which forms the dark matter. It gives a hope, supported by the calculations done so far [4, 7–9, 24, 25], that we shall understand the differences in properties of family members.

The theory also predicts that since there are several scalar fields, which mass eigen states differ from the superposition with which they couple to different family members of different

families and also to weak bosons, future experiments will observe several scalar fields.

II. PROPERTIES OF FERMIONS, GAUGE VECTOR AND SCALAR BOSONS

The starting symmetry $SO(13,1)$ breaks into $SO(7,1) \times SU(3) \times U(1)_{II}$ and then to $SO(3,1) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$. In this paper we follow mainly the stage just before the electroweak break and after the electroweak break. We shall call the break from $SU(2)_I \times SU(2)_{II} \times U(1)_{II}$ to $SU(2)_I \times U(1)_I$ the *break II* and the break from $SU(2)_I \times U(1)_I$ to $U(1)$, that is the electroweak break, the *break I*.

The *spin-charge-family* theory [1–9, 24–27], a kind of the Kaluza-Klein-like theory but with families included, proposes in $d = (13 + 1)$ a simple action for a Weyl spinor and for the corresponding gauge fields

$$\begin{aligned}
S &= \int d^d x \mathcal{L}_f + \int d^d x E (\alpha R + \tilde{\alpha} \tilde{R}), \\
\mathcal{L}_f &= \frac{1}{2} (E \bar{\psi} \gamma^a p_{0a} \psi) + h.c., \\
p_{0a} &= f^\alpha_a p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^\alpha_a\}_-, \quad p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}, \\
R &= f^{\alpha[a} f^{\beta b]} (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \omega^c_{b\beta}), \quad \tilde{R} = f^{\alpha[a} f^{\beta b]} (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^c_{b\beta}). \quad (1)
\end{aligned}$$

The spin connection fields $\omega_{ab\alpha}$ are the gauge fields of the "charges" $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$, while $\tilde{\omega}_{ab\alpha}$ are the gauge fields of the "family" quantum numbers $\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$.

The fermion part of the action manifests after the breaks of symmetries to $SO(3,1) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ as

$$\begin{aligned}
\mathcal{L}_f &= \bar{\psi} \gamma^n (p_m - \sum_{A,i} g^A \tau^{Ai} A_m^{Ai}) \psi + \\
&\sum_{s=7,8} \bar{\psi} \gamma^s (p_s - \sum_{\tilde{A},i} \tilde{g}^{\tilde{A}} \tilde{\tau}^{\tilde{A}i} \tilde{A}_s^{\tilde{A}i} - \sum_{A,i} g^A \tau^{Ai} A_s^{Ai}) \psi + \\
&\text{the rest}, \quad (2)
\end{aligned}$$

where $n = 0, 1, 2, 3$ and

$$\begin{aligned}
\tau^{Ai} &= \sum_{a,b} c^{Ai}_{ab} S^{ab}, \quad \{\tau^{Ai}, \tau^{Bj}\}_- = i \delta^{AB} f^{Aijk} \tau^{Ak}, \\
\tilde{\tau}^{Ai} &= \sum_{a,b} \tilde{c}^{Ai}_{ab} \tilde{S}^{ab}, \quad \{\tilde{\tau}^{Ai}, \tilde{\tau}^{Bj}\}_- = i \delta^{AB} f^{Aijk} \tilde{\tau}^{Ak}. \quad (3)
\end{aligned}$$

τ^{Ai} determine all the charges of fermions, A_m^{Ai} , $m \in \{0, 1, 2, 3\}$; (they are superposition of $f_a^\alpha \omega_{bc\alpha}$) the corresponding gauge vector fields and A_s^{Ai} , $s \in \{7, 8\}$; the gauge scalar fields (superposition of $f_s^\sigma \omega_{t\sigma}$), while $\tilde{\tau}^{Ai}$ determine all the family quantum numbers (the family charges) of fermions and \tilde{A}_s^{Ai} (superposition of $f_a^\sigma \tilde{\omega}_{bc\sigma}$) the corresponding gauge scalar fields. The way of breaking symmetries is assumed in a way that it leads to low energy observable phenomena.

At this step, that is before the *break II*, the action describes eight massless fermions, interacting with the i. massless vector triplet ($SU(2)_{II}$) gauge fields, massless vector triplet ($SU(2)_I$) weak gauge fields, massless vector singlet ($U(1)_{II}$) gauge fields, massless vector octet ($SU(3)$) gauge fields, and ii. scalar fields of both kinds, that is of S^{ab} and of \tilde{S}^{ab} origin. The term vectors and scalars determines the relation to $d = (3 + 1)$.

The scalar fields of any origin, appearing in the second line of Eq. (2), which would manifest as four vectors with respect to $s = 5, 6, 7, 8$, manifest as doublets for $s = 7, 8$. Namely, if s would be allowed in Eq. (2) to run within all the indices of $SO(4)$, that is, if $s \in \{5, 6, 7, 8\}$, the scalar fields would be in the vector representation of the group $SO(4)$: $(\frac{1}{2}, \frac{1}{2})$. The choice of $s \in \{7, 8\}$ forces them to behave as doublets with respect to the $SU(2)_I$, that is with respect to the weak subgroup. Correspondingly all the scalar fields behave as doublets with respect to the weak charge, while they are triplets with respect to the family quantum numbers, generated by \tilde{S}^{ab} .

There are scalar fields (\vec{A}_s^2 , $\vec{A}_s^{\tilde{N}_R}$, $s = 7, 8$) which, after gaining nonzero vacuum expectation values in the *break II*, determine masses of the upper four families and of the gauge bosons $A_m^{2\pm} = \frac{1}{\sqrt{2}} (A_m^{21} \mp iA_m^{22})$, $A_m^{Y'} = -\sin \theta_2 A_m^4 + \cos \theta_2 A_m^{23}$, while the gauge vector fields $A_m^Y = \cos \theta_2 A_m^4 + \sin \theta_2 A_m^{23}$ and \vec{A}_m^1 and the lower four families remain massless (see footnote [34] for definitions of quantum numbers).

In the electroweak *break I* the scalar fields (\vec{A}_s^1 , $\vec{A}_s^{\tilde{N}_L}$, A_s^Q , $A_s^{Q'}$ and $A_s^{Y'}$, $s = 7, 8$) gain nonzero vacuum expectation values, determining masses of the lower four families and of the gauge bosons $A_m^{1\pm} = \frac{1}{\sqrt{2}} (A_m^{11} \mp iA_m^{12})$, $A_m^{Q'} = -\sin \theta_1 A_m^Y + \cos \theta_1 A_m^{13}$, while $A_m^Q = \cos \theta_1 A_m^Y + \sin \theta_1 A_m^{13}$ remains massless (see footnote [35] for definitions of quantum numbers).

Let us denote the scalar fields contributing to masses of the lower four families (scalar fields, contributing to the *break II* are presented in the appendix in Eq. (A.1)) and to the

weak bosons masses with a common vector

$$\begin{aligned}\Phi^{I Ai} &\equiv \Phi_{\mp}^{I Ai}, \quad \Phi_{\mp}^{I Ai} = (\vec{A}_{\mp}^1, \vec{A}_{\mp}^{\tilde{N}_L}, A_{\mp}^{Y'}, A_{\mp}^{Q'}, A_{\mp}^Q), \\ \Phi_{\mp}^{Ai} &= (\Phi_7^{Ai} \pm i\Phi_8^{Ai}), \quad A_I = \{\tilde{1}, \tilde{N}_L, Y', Q', Q\}.\end{aligned}\quad (4)$$

We choose a renormalizable effective potential $V(\Phi^{I, Ai})$ for the (assumed to be) real scalar fields $\Phi^{I, Ai}$ (Eq. (4)), which couple among themselves

$$V(\Phi^{I Ai}) = \sum_{A,i} \left\{ -\frac{1}{2} (m_{Ai}^I)^2 (\Phi^{I Ai})^2 + \frac{1}{4} \sum_{B,j} \lambda^{I Ai Bj} (\Phi^{I Ai})^2 (\Phi^{I Bj})^2 \right\}. \quad (5)$$

Couplings among the scalar fields are here chosen to be symmetric: $\lambda^{Ai Bj} = \lambda^{Bj Ai}$.

Scalar fields couple to the gauge bosons at the *break* I according to the Lagrange function \mathcal{L}_{sI}

$$\begin{aligned}\mathcal{L}_{sI} &= \sum_{A,i} (p_{0m} \Phi^{I Ai})^\dagger (p_0^m \Phi^{I Ai}) - V(\Phi^{I Ai}), \\ p_{0m} &= p_m - \{g^Y \tau^Y A_m^Y + g^1 \vec{\tau}^1 \vec{A}_m^1\}.\end{aligned}\quad (6)$$

We shall study properties of scalar fields in subsection II A and section III.

Expressing the operators γ^7 and γ^8 in terms of the nilpotents $(\pm)^{\tilde{78}}$ (appendix), the second line of Eq. (2) can be rewritten as follows

$$\begin{aligned}\bar{\psi} M \psi &= \sum_{s=7,8} \bar{\psi} \gamma^s p_{0s} \psi = \psi^\dagger \gamma^0 ((-)^{\tilde{78}} p_{0-} + (+)^{\tilde{78}} p_{0+}) \psi, \\ (\pm)^{\tilde{78}} &= \frac{1}{2} (\gamma^7 \pm i\gamma^8), \\ p_{0\pm} &= (p_{07} \mp i p_{08}), \quad p_{0\pm} = p_{\pm} - \frac{1}{2} S^{ab} \omega_{ab\pm} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\pm} \\ \omega_{ab\pm} &= (f_7^\sigma \mp i f_8^\sigma) \omega_{ab\sigma}, \quad \tilde{\omega}_{ab\pm} = (f_7^\sigma \mp i f_8^\sigma) \tilde{\omega}_{ab\sigma}.\end{aligned}\quad (7)$$

In the mass term $\bar{\psi} M \psi$ there is $(-)^{\tilde{78}}$ which transforms a weak chargeless u_R^i (ν_R^i) with $Y = \frac{2}{3}$ ($Y = -\frac{1}{2}$) into the corresponding left handed members and it is $(+)^{\tilde{78}}$ which transforms a weak chargeless d_R^i (e_R^i) with $Y = -\frac{1}{3}$ ($Y = -\frac{1}{2}$) into the corresponding left handed members for each family separately. And there are $\vec{\tau}^A$ which transform a family member of one family to the same family member of another family.

There are eight massless families before the *break* II . Four families out of eight are doublets with respect to $\vec{\tau}^2$ (one $\tilde{S}U(2)$ subgroup of $\tilde{S}O(4)$, see footnotes for the definition)

and \vec{N}_R (one $\tilde{S}U(2)$ subgroup of $\tilde{S}O(3,1)$, see footnotes for the definition). The rest four families are doublets with respect to (the rest $\tilde{S}U(2)$ subgroup of $\tilde{S}O(4)$) $\vec{\tau}^1$ and (the rest $\tilde{S}U(2)$ subgroup of $\tilde{S}O(3,1)$) \vec{N}_L .

The first group of four families become massive, when the gauge scalar fields (the superposition of the scalar fields $\tilde{\omega}_{abs}$) of the charges $\vec{\tau}^2$ and \vec{N}_R gain nonzero vacuum expectation values. Since family quantum numbers do not distinguish among family members, the masses of u^{IIi} , d^{IIi} , ν^{IIi} and e^{IIi} are the same on the tree level, if $\Phi_+^{IIAi} = \Phi_-^{IIAi}$.

The second group of four families become massive, when gauge scalar fields (superposition of $\tilde{\omega}_{abs}$) of the charges $\vec{\tau}^1$ and \vec{N}_L , together with those (superposition of $\omega_{sts'}$) of the $U(1)$ charges (Y', Q', Q) gain nonzero vacuum expectation values.

The mass term $\bar{\psi} M \psi$ determines in this case the tree level mass matrices of quarks and leptons of the lower four families, among which are the measured three families, and correspondingly the masses, the Yukawa couplings and the mixing matrices for these four families after the loop corrections are taken into account. The tree level mass term distinguishes among the members of any of the four families: Due to the eigen values of the operators Q, Q' and Y' , and due to $\binom{78}{\mp} \Phi^{IAi}$, since (by assumption) $\Phi_+^{IAi} \neq \Phi_-^{IAi}$. The fields $\tilde{A}_{\mp}^{\tilde{N}_L i}$ and \tilde{A}_{\mp}^{1i} are triplets with respect to the two $\tilde{S}U(2)$ family subgroups $(\vec{\tau}^1, \vec{N}_L)$, while they are doublets with respect to the weak charge (see subsection II A).

After the electroweak break the effective Lagrange density for the lower four families of fermions looks [3] as (Eqs. (2, 7))

$$\begin{aligned}
\mathcal{L}_{If} &= \bar{\psi} (\gamma^m p_{0m} - ((-)^{78} p_{0-} + (+)^{78} p_{0+})) \psi, \\
p_{0m} &= p_m - \{ g^Q Q A_m + g^{Q'} Q' Z_m^{Q'} + \frac{g^1}{\sqrt{2}} (\tau^{1+} W_m^{1+} + \tau^{1-} W_m^{1-}) + g^{Y'} Y' A_m^{Y'} \} \\
p_{0\pm} &= p_{\pm} - \{ \tilde{g}^{\tilde{N}_L} \vec{N}_L \cdot \vec{\tilde{A}}_{\pm}^{\tilde{N}_L} + \tilde{g}^{\tilde{Q}'} \tilde{Q}' \cdot \vec{\tilde{A}}_{\pm}^{\tilde{Q}'} + \frac{\tilde{g}^1}{\sqrt{2}} (\tilde{\tau}^{1+} \tilde{A}_{\pm}^{1+} + \tilde{\tau}^{1-} \tilde{A}_{\pm}^{1-}) \\
&\quad + g^Q Q A_{\pm}^Q + g_1^{Q'} Z_{\pm}^{Q'} + g_1^{Y'} Y' A_{\pm}^{Y'} \}, \\
Q &= \tau^{13} + Y, \quad Q'_1 = (\tau^{13} - \tan^2 \theta_1 Y), \quad Y'_1 = (\tau^{23} - \tan^2 \theta_2 \tau^4). \tag{8}
\end{aligned}$$

Q and Q' are the *standard model* like charges (ϑ_1 does not need to be θ_1), Y' is the additional quantum number [3], appearing in the *spin-charge-family* theory (similarly as it does in the $SO(10)$ models), and $g^Q = g^Y \cos \theta_1$, $g^{Q'} = g^1 \cos \theta_1$ and $g^{Y'} = g^4 \cos \theta_2$.

A. Transformation properties of scalar fields

In the *standard model* the Higgs and the anti-Higgs (a kind of Majorana) "dress" the right handed fermions with the appropriate weak and hypercharge so that the "dressed" ones carry quantum numbers of the left handed partners.

How can these properties of the Higgs be explained in the *spin-charge-family* theory? It is the second summand in Eq. (2) which determines on the tree level, after scalar fields gain non zero vacuum expectation values, masses of fermions. In the case that all four indices $s = (5, 6, 7, 8)$ would be allowed, all the scalar fields would transform as vectors (as well known from the vector representations in $d = (3+1)$) according to the $(\frac{1}{2}, \frac{1}{2})$ representation. Since only $s = (7, 8)$ is allowed, they transform as doublets with respect to the weak $SU(2)$.

We also notice that the operators $\gamma^0 \begin{pmatrix} 78 \\ \mp \end{pmatrix} \tau^{Ai} \Phi_{\mp}^{Ai}$ transform right handed weak chargeless fermions of a particular hyper charge into the corresponding left handed weak charged partners

$$\begin{aligned} \gamma^0 \begin{pmatrix} 78 \\ - \end{pmatrix} \tau^{Ai} \Phi_{-}^{Ai} \psi_{(u,\nu)R} &\rightarrow \tau^{Ai} \Phi_{-}^{Ai} \psi_{(u,\nu)L} , \\ \gamma^0 \begin{pmatrix} 78 \\ + \end{pmatrix} \tau^{Ai} \Phi_{+}^{Ai} \psi_{(d,e)R} &\rightarrow \tau^{Ai} \Phi_{+}^{Ai} \psi_{(d,e)L} . \end{aligned} \quad (9)$$

Here τ^{Ai} stay for all the quantum numbers of either \tilde{S}^{ab} (in the case of the lower four families these are $\vec{\tau}^1$ and \vec{N}_L , causing transitions among families) or S^{ab} (Y', Q' and Q , which are diagonal: In the family member and the family quantum numbers), while Φ_{\mp}^{Ai} (Eq. (4)) stay for all the corresponding scalar gauge fields of either $\tilde{\omega}_{st\mp}$ or $\omega_{st\mp}$ origin.

Eq. (9) demonstrates that all the scalar fields transform as doublets with respect to the weak charge, each of the two kinds $(\Phi_{-}^{Ai}, \Phi_{+}^{Ai})$ having different hyper charges (Φ_{-}^{Ai} is a member of a doublet with the weak charge $\frac{1}{2}$ and the hypercharge $-\frac{1}{2}$, while Φ_{+}^{Ai} is a member of a doublet with the weak charge $-\frac{1}{2}$ and the hypercharge $\frac{1}{2}$), the same for all the families. Their nonzero vacuum expectation values do not need to be the same: ($\langle \Phi_{-}^{Ai} \rangle \neq \langle \Phi_{+}^{Ai} \rangle$, for any (A, i)). In this case the pair with nonzero vacuum expectation values does not look like a Majorana.

Scalar fields which change properties of the upper four families do not distinguish on the tree level among family members provided that $\langle \vec{A}_{-}^2 \rangle = \langle \vec{A}_{+}^2 \rangle$ and $\langle \vec{A}_{-}^{\tilde{N}_R} \rangle = \langle \vec{A}_{+}^{\tilde{N}_R} \rangle$ for each vector component. In this case all the family members of a particular family have on the tree level the same mass. They keep correspondingly the $SU(2)_I$ (the weak charge)

symmetry unchanged, although the right handed partners (having the same mass as the left handed ones) do not carry the weak charge.

The scalar fields ($\vec{A}_\pm^1, \vec{A}_\pm^{\tilde{N}_L}$, all transforming as doublets with respect to the weak charge group) transform as triplets with respect to the $\tilde{SU}(2)$ groups (the generators of which are $\vec{\tau}^1, \vec{N}_L$), while they transform as singlets with respect to $U(1)$ groups (the generators of which are $Y', Q',$ and Q).

Scalar fields with nonzero vacuum expectation values obviously change properties of the vacuum. There are several changes of the vacuum during several breaks of symmetries of the starting one: $SO(13,1)$. Let us concentrate on *break I*. The *break II* is discussed in appendix .

At the electroweak *break I* the vacuum changes again. The mass term (Eq. (7)) forces all the scalar fields, which obtain nonzero vacuum expectation values at this break, to be doublets with respect to the $SU(2)_I$ (weak) charge, while they keep their adjoint representations with respect to all the charges of the \tilde{S}^{ab} origin ($\vec{\tau}^1, \vec{N}_L$). To the vacuum two new kinds of terms must be added

$$\begin{aligned} (-) \ominus_I &= (-) T s_{\tilde{N}_L} |(+) T d_{(-)\vec{\tau}^1} || \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ [+ & + & + & + & + & + \end{smallmatrix} , \\ (+) \oplus_I &= (+) T s_{\tilde{N}_L} |(-) T d_{(+)\vec{\tau}^1} || \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ [- & - & - & - & - & - \end{smallmatrix} . \end{aligned} \quad (10)$$

Here $T s_{\tilde{N}_L}$ denotes a triplet with respect to the operators \vec{N}_L and a singlet with respect to \vec{N}_L , while $(\begin{smallmatrix} 56 & 78 \\ [+ & + \end{smallmatrix}) T d_{(\mp)\vec{\tau}^1}$ are the two triplets with respect to $\vec{\tau}^1$ and doublets with respect to $\vec{\tau}^1$.

One finds

$$\begin{aligned} \tau^{1+} \tau^{1-} \begin{pmatrix} 78 \\ (+) \oplus_I \end{pmatrix} &= \begin{pmatrix} 78 \\ (+) \oplus_I \end{pmatrix}, \quad \tau^{1-} \tau^{1+} \begin{pmatrix} 78 \\ (-) \ominus_I \end{pmatrix} = \begin{pmatrix} 78 \\ (-) \ominus_I \end{pmatrix}, \\ Q \begin{pmatrix} 78 \\ (+) \oplus_I \end{pmatrix} &= 0 = Q \begin{pmatrix} 78 \\ (-) \ominus_I \end{pmatrix}, \\ Q' \begin{pmatrix} 78 \\ (+) \oplus_I \end{pmatrix} &= -\frac{1}{2 \cos^2 \theta_1}, \quad Q' \begin{pmatrix} 78 \\ (-) \ominus_I \end{pmatrix} = \frac{1}{2 \cos^2 \theta_1}. \end{aligned} \quad (11)$$

We see that in the *break I* the fields $A_m^{1\pm} (= W_m^\pm)$ and $A_m^{Q'} (= Z_m) = \cos \theta_1 A_m^{13} - \sin \theta_1 A_m^Y$ become massive, while $A_m^Q (= A_m) = \sin \theta_2 A_m^{13} + \cos \theta_1 A_m^Y$ stays massless, provided that $\frac{g_1}{g_Y} \tan \theta_1 = 1$.

In the ref. [8] properties of the stable fifth family members were studied and the possibility that they form a dark matter analysed. We study in this paper properties of scalar and vector gauge fields and of families of quarks and leptons for (mostly) the lower four families.

III. SCALAR FIELDS IN MINIMIZATION PROCEDURE ON TREE LEVEL

Let us look for the minimum of the two potentials presented in Eq. (5) and search for the mass eigen states on the tree level [5]. Let Φ^{Ai} denotes $\Phi^{I Ai}$ (or $\Phi^{II Ai}$) and $V(\Phi^{Ai})$ the corresponding effective potential. First we look for the first derivatives with respect to all the interacting scalar fields and put them equal to zero

$$\frac{\partial V(\Phi^{Ai})}{\partial \Phi^{Ai}} = 0 = \Phi^{Ai} [-(m_{Ai})^2 + \lambda^{Ai}(\Phi^{Ai})^2 + \sum_{B,j} \lambda^{Ai Bj} (\Phi^{Bj})^2]. \quad (12)$$

Here the notation $\lambda^{Ai} := \lambda^{Ai Ai}$ is used. When expressing the minimal values of the scalar fields, let us call them v_{Ai} , as functions of the parameters, Eq. (12) leads to the coupled equations for the same number of unknowns $v_{Ai} = \Phi_{min}^{Ai}$

$$-(m_{Ai})^2 + \sum_{B,j} \lambda^{Ai Bj} (v_{Bj})^2 = 0. \quad (13)$$

Looking for the second derivatives at the minimum determined by v_{Ai} one finds

$$\frac{\partial^2 V(\Phi^{Ck})}{\partial \Phi^{Ai} \partial \Phi^{Bj}} \Big|_{v_{Ck}} = 2\lambda^{Ai Bj} v_{Ai} v_{Bj}. \quad (14)$$

Let us look for the basis Φ^ρ (we should keep in mind the index (\pm) , although we do not write it down)

$$\Phi^{Ai} = \sum_{\rho} \mathcal{C}_{\rho}^{Ai} \Phi^{\rho}, \quad (15)$$

in which on the tree level the potential would be diagonal

$$V(\Phi^{\rho}) = \sum_{\rho} \left\{ -\frac{1}{2} (m_{\rho})^2 (\Phi^{\rho})^2 + \frac{1}{4} \lambda^{\rho} (\Phi^{\rho})^4 \right\}, \quad (16)$$

with $\frac{\partial V}{\partial \Phi^{\rho}} \Big|_{v_{Ai}} = \sum_{Ai} \frac{\partial V}{\partial \Phi^{Ai}} \frac{\partial \Phi^{Ai}}{\partial \Phi^{\rho}} \Big|_{v_{Ai}} = 0$, with Φ_{min}^{ρ} at these points called $v_{\rho} = \sum_{Ai} \mathcal{C}_{\rho}^{Ai T} v_{Ai}$ (T denotes transposition), and correspondingly with $\frac{\partial^2 V}{\partial (\Phi^{\rho})^2} \Big|_{v_{\rho}} = -(m_{\rho})^2 + 3\lambda^{\rho} (\Phi^{\rho})^2 \Big|_{v_{\rho}} = 2\lambda^{\rho} (v_{\rho})^2 = \sum_{A,i,B,k} 2\lambda^{Ai Bj} v_{Ai} v_{Bj} \mathcal{C}_{\rho}^{Ai} \mathcal{C}_{\rho}^{Bj}$. This means that the new basis can be found by diagonalizing the matrix of the second derivatives at the minimum and correspondingly put to zero the determinant

$$\det \begin{pmatrix} 2\lambda^{\tilde{N}_L 1} (v_{\tilde{N}_L 1})^2 - 2(m_{\rho})^2, & 2\lambda^{\tilde{N}_L 1 \tilde{N}_L 2} v_{\tilde{N}_L 1} v_{\tilde{N}_L 2}, & 2\lambda^{\tilde{N}_L 1 \tilde{N}_L 3} v_{\tilde{N}_L 1} v_{\tilde{N}_L 3}, & \dots \\ 2\lambda^{\tilde{N}_L 2 \tilde{N}_L 1} v_{\tilde{N}_L 2} v_{\tilde{N}_L 1}, & 2\lambda^{\tilde{N}_L 2} (v_{\tilde{N}_L 2})^2 - 2(m_{\rho})^2, & 2\lambda^{\tilde{N}_L 2 \tilde{N}_L 3} v_{\tilde{N}_L 2} v_{\tilde{N}_L 3}, & \dots \\ \vdots & \vdots & \ddots & \end{pmatrix}. \quad (17)$$

The same number of orthogonal scalar fields Φ^β , with nonzero vacuum expectation values and nonzero masses, as we started with, follow. To each of them one eigen value $2(m_\rho)^2$ corresponds, determined by the parameters m_{Ai} and λ^{AiBj} of Eq. (5).

For the time evolution of the free scalar fields one correspondingly finds for each β

$$\Phi^\beta(t) = e^{-im_\beta(t-t_0)} \Phi^\beta(t_0). \quad (18)$$

A. A simple example

Let us examine a simple case, one triplet, say \vec{A}^1 , and let us call these three scalar states Φ^i . Following Eq. (12) one obtains $-(m_i)^2 + \sum_j \lambda^{ij}(v_j)^2 = 0$, for each $i = 1, 2, 3$. Let us further simplify the example by the assumption that one of these three fields is decoupled: $\lambda^{i3} = 0$, for $i = (1, 2)$. Then it follows for the vacuum expectation values $v_i, i \in \{1, 2, 3\}$

$$(v_1)^2 = \frac{-\lambda^{12}(m_2)^2 + \lambda^2(m_1)^2}{\lambda^1\lambda^2 - (\lambda^{12})^2}, \quad (v_2)^2 = \frac{-\lambda^{12}(m_1)^2 + \lambda^1(m_2)^2}{\lambda^1\lambda^2 - (\lambda^{12})^2}, \quad (v_3)^2 = \frac{(m_3)^2}{\lambda^3}. \quad (19)$$

The second derivatives at the minimum, $\frac{\partial^2 V(\Phi^k)}{\partial \Phi^i \partial \Phi^j} |_{\Phi^k=v_k} = 2\lambda^{ij}v_i v_j$, lead to the determinant (Eq.(17)), from where one obtains the eigen masses

$$(m^{1,2})^2 = \frac{1}{2} \{ [\lambda^1(v_1)^2 + \lambda^2(v_2)^2] \mp \sqrt{[\lambda^2(v_2)^2 - \lambda^1(v_1)^2]^2 + 4(\lambda^{12})^2(v_1)^2(v_2)^2} \}, \quad (20)$$

and $(m^3)^2 = (m_3)^2$. If the coupling between the two scalar components is zero, the trivial case of three uncoupled scalar fields follows. In the case that the two masses, m_1 and m_2 , are equal and that also the two self strengths are the same, $\lambda^1 = \lambda^2$, then $(v_1)^2 = (v_2)^2$ and the two eigen values for masses are $(m^{1,2})^2 = (v_1)^2[(\lambda^1 - \lambda^{12}), (\lambda^1 + \lambda^{12})]$. In the case that λ^1 and λ^{12} are close to each other, the two eigen values differ a lot. In the case of $\lambda^{12} = 0$ the two scalars would manifest and be observed as only one.

Such a simplified situation illustrates that the mass eigen states of the scalar fields might differ a lot from the superposition of the scalar fields which couples to any of the family members of any of the families, the tree level mass matrices of which are presented in Table I and in Eq. (21) and discussed in next section IV.

IV. COUPLING OF FAMILY MEMBERS TO SCALAR FIELDS

There is only explanation for the dark matter [8] as the clusters of the stable fifth family members which supports so far the existence of the upper four families. To differences in

i	1	2	3	4
1	$-\frac{1}{2}(\tilde{a}_{\mp}^{13} + \tilde{a}_{\mp}^{\tilde{N}_L^3})$	$\tilde{a}_{\mp}^{\tilde{N}_L^-}$	0	\tilde{a}_{\mp}^{1-}
2	$\tilde{a}_{\mp}^{\tilde{N}_L^+}$	$\frac{1}{2}(-\tilde{a}_{\mp}^{13} + \tilde{a}_{\mp}^{\tilde{N}_L^3})$	\tilde{a}_{\mp}^{1-}	0
3	0	\tilde{a}_{\mp}^{1+}	$\frac{1}{2}(\tilde{a}_{\mp}^{13} - \tilde{a}_{\mp}^{\tilde{N}_L^3})$	$\tilde{a}_{\mp}^{\tilde{N}_L^-}$
4	\tilde{a}_{\mp}^{1+}	0	$\tilde{a}_{\mp}^{\tilde{N}_L^+}$	$\frac{1}{2}(\tilde{a}_{\mp}^{13} + \tilde{a}_{\mp}^{\tilde{N}_L^3})$

TABLE I: The contributions of the fields $(-\tilde{g}^1 \tilde{\tau}^1 \tilde{A}_{\mp}^1, -\tilde{g}^{\tilde{N}_L} \tilde{N}_L^i \tilde{A}_{\mp}^{\tilde{N}_L})$ to the mass matrices on the tree level ($\mathcal{M}_{(o)}$) for the lower four families of quarks and leptons after the electroweak break are presented. The notation $\tilde{a}_{\mp}^{\tilde{A}i} = -\tilde{g}^{\tilde{A}} \tilde{v}_{\tilde{A}i\mp}$ is used.

masses of the family members of the lower four families contribute on the tree level not only the differences in the expectation values $\langle \Phi_{-}^{IAi} \rangle \neq \langle \Phi_{+}^{IAi} \rangle$, $\Phi_{\mp}^{IAi} = (\tilde{A}_{\mp}^1, \tilde{A}_{\mp}^{\tilde{N}_L})$, but also the scalar fields $(A_{\mp}^{Y'}, A_{\mp}^{Q'}, A_{\mp}^Q)$, which couple to each member of a family in a different way.

The tree level contributions of $\tilde{A}_{\mp}^{\tilde{N}_L}$ and \tilde{A}_{\mp}^1 (Eq. (8)) to the mass matrix of any family member look [3, 24] as it is presented in Table I. The notation $\tilde{a}_{\mp}^{\tilde{A}i} = -\tilde{g}^{\tilde{A}} \tilde{v}_{\tilde{A}i\mp}$ is used, where $\tilde{v}_{\tilde{A}i\mp}$ are the vacuum expectation values of the corresponding scalars ($\langle \tilde{A}_{\mp}^{Ai} \rangle$). Let us repeat that $\tilde{a}_{\mp}^{\tilde{A}i}$ distinguish among (u^i, ν^i) $((-))$ and (d^i, e^i) $((+))$.

The contributions of $g_1^Q, Q A_{\mp}^Q, g_1^{Q'} Q' Z_{\mp}^{Q'}$, and $g^{Y'} Y' A_{\mp}^{Y'}$ are not presented in Table I. They are different for each of the family member $\alpha = (u^i, d^i, \nu^i, e^i)$ and the same for all the families ($i = (1, 2, 3, 4)$)

$$a_{\mp}^{\alpha} = -\{g_1^Q Q^{\alpha} v_{Q\mp} + g_1^{Q'} Q'^{\alpha} v_{Q'\mp} + g^{Y'} Y'^{\alpha} v_{Y'\mp}\}, \quad (21)$$

with Q^{α} , Q'^{α} and Y'^{α} , which are eigen values of the corresponding operators for the family member state α .

While the mass matrix elements might (and are expected to) change considerably when higher order corrections are taken into account, we expect that the symmetries stay the ones of the tree level contributions, the same for all the family members, as presented on Table I. Loop corrections, to which also the massive gauge fields and dynamical massive scalar fields contribute, are expected to strongly influence fermions properties. These calculations are in progress [9, 24] and look so far very promising in offering the right answers for the masses and mixing matrices of all the family members: quarks and leptons, treating colour charge and colour chargeless members in an equivalent way. It turns out that both, quarks and

leptons, mass matrices behave in a very similar way. No additional neutrinos, offering a "sea-saw" mechanism, are needed.

Let $\psi_{(L,R)}^\alpha$ denote massless and $\Psi_{(L,R)}^\alpha$ massive four vectors for each family member $\alpha = (u_{L,R}, d_{L,R}, \nu_{L,R}, e_{L,R})$ after taking into account loop corrections in all orders [3, 24], $\psi_{(L,R)}^\alpha = V_{(L,R)}^\alpha \Psi_{(L,R)}^\alpha$, and let $(\psi_{(L,R)}^{\alpha k}, \Psi_{(L,R)}^{\alpha k})$ be any component of the four vectors, massless and massive, respectively. On the tree level we have $\psi_{(L,R)}^\alpha = V_{(o)}^\alpha \Psi_{(L,R)}^{\alpha(o)}$ and

$$\langle \psi_L^\alpha | \gamma^0 \mathcal{M}_{(o)}^\alpha | \psi_R^\alpha \rangle = \langle \Psi_L^{\alpha(o)} | \gamma^0 V_{(o)}^{\alpha\dagger} \mathcal{M}_{(o)}^\alpha V_{(o)}^\alpha | \Psi_R^{\alpha(o)} \rangle, \quad (22)$$

with $\mathcal{M}_{(o)kk'}^\alpha = \sum_{A,i} (-g^{Ai} v_{Ai\mp}) C_{kk'}^\alpha$. The coefficients $C_{kk'}^\alpha$ can be read from Table I. It then follows

$$\begin{aligned} \bar{\Psi}^\alpha V_{(o)}^{\alpha\dagger} \mathcal{M}_{(o)}^\alpha V_{(o)}^\alpha \Psi^\alpha &= \bar{\Psi}^\alpha \text{diag}(m_{(o)1}^\alpha, \dots, m_{(o)4}^\alpha) \Psi^\alpha, \\ V_{(o)}^{\alpha\dagger} \mathcal{M}_{(o)}^\alpha V_{(o)}^\alpha &= \Phi_{f(o)}^\alpha. \end{aligned} \quad (23)$$

The coupling constants $m_{(o)k}^\alpha$ (in some units) of the dynamical scalar fields $\Phi_{f(o)k}^\alpha$ to the family member $\Psi^{\alpha k}$ belonging to the k^{th} family are on the tree level correspondingly equal to

$$(\Phi_{\Psi(o)}^\alpha)_{kk'} \Psi^{\alpha k'} = \delta_{kk'} m_{(o)k}^\alpha \Psi^{\alpha k}. \quad (24)$$

The superposition of scalar fields $(\Phi_{f(o)}^\alpha)$, which couple to fermions [36] and depend on the quantum numbers α and k , are in general different from the superposition Φ^β (Eqs. (15,18)), which are the mass eigen states. Each family member α of each massive family k couples in general to different superposition of scalar fields.

The two kinds of superposition are expressible with each other

$$\Phi_{f(o)k}^\alpha = \sum_\beta D_k^{\alpha\beta} \Phi^\beta. \quad (25)$$

V. SCALARS WHICH BRING MASSES TO WEAK BOSONS

In the *break I* to the vacuum of till this step two additional terms must be added (Eq.(10)), namely $\overset{78}{(-)} \ominus_I + \overset{78}{(+)} \oplus_I$, while Q of this vacuum is equal to zero. We obtain for the mass term of the weak vector bosons, which interact with the scalar fields responsible for the appearance the *break I*, as follows

$$\left(\frac{1}{2}\right)^2 (g^1)^2 v_I^2 \left(\frac{1}{(\cos \theta_1)^2} Z_m^{Q'} Z^{Q'm} + 2 W_m^+ W^{-m} \right). \quad (26)$$

To the vacuum expectation value v_I all the scalar fields Φ_{\mp}^{IAi} contribute. (These gauge vector bosons do not couple to the scalar doublets which cause the appearance of the *break II*.)

Let us conclude this section by recognizing that the contributions to masses of vector bosons are on the tree level determined by the coupling constants and the vacuum expectation values of these fields.

VI. CONCLUSIONS

It is demonstrated (on the tree level) that according to the *spin-charge-family* theory – which predicts the families of fermions and their charges, the gauge fields and several scalar fields – each family member $\Psi^{\alpha k}$ ($\alpha = (u, d, \nu, e)$) of each family ($i = (1, 2, 3, 4)$) couples to a different superposition of the scalar fields $\Phi_{f(0)}^{\alpha}$, Eq. (23), with the coupling constant proportional to its (fermion) mass (Eq. (24)). Each of these superposition differs from the superposition of the scalar fields which determine masses of gauge vector bosons (Eqs. (26, A.6)), which further differ from the mass eigen states of scalars (Eq. (18)).

At each break the scalar fields gaining nonzero vacuum expectation values change the vacuum (Eqs.(9, 10)). At the electroweak break (*break I*) the scalar fields, forbidden to behave as a vector in the $(\frac{1}{2}, \frac{1}{2})$ representation, behave as doublets (that is manifesting the fundamental representation) with respect to the weak $SU(2)$ group, while they behave like triplets with respect to the family groups ($\tilde{\tau}^{1i}$ and \tilde{N}_L^i) and as singlets with respect to several $U(1)$ with the generators (Y', Q', Q) . At the electroweak break the scalar fields, which determine fermion masses on the tree level, take care of the differences in the masses among family members: i. Through differences in their vacuum expectation values when they couple to (u^i, ν^i) and when they couple to (d^i, e^i) , and ii. Through differences in the eigen values of the operators (Y', Q', Q) appearing with the corresponding scalar fields. Properties depend on the parameters, the values of which are in this paper not discussed. The fermion mass matrices manifest symmetries which limit strongly free parameters of the theory.

Since the mass term (Eq.(7)) explains why the scalar fields behave like doublets with respect to the weak $SU(2)$ group, while they keep the character of the adjoint representations with respect to the family groups, the *standard model* can really be interpreted as a low

energy effective manifestation of the *spin-charge-family* theory.

Let me say that I assumed (make a choice among the possibilities) such breaks of symmetries of a simple starting action which lead to observable phenomena. It is, however, far from saying that I move the assumptions of the *standard model* to next step, since many assumptions of the *standard model* get explanations through the *spin-charge-family* theory.

Let me conclude the paper with the predictions of the *spin-charge-family* theory: Observations of the scalar fields at the LHC and other experiments are expected to differ from the predictions of the *standard model*, although so far the experimental data have shown no disagreement with the *standard model* predictions. Several scalar fields, predicted by the theory, will show up. The fourth family will sooner or later be observed and it might be that the observed ten-jet event is caused by the fourth family members. There will be no supersymmetric partners.

A systematic study of predictions, which would take into account the loop corrections of the *spin-charge-family* theory is needed and it is in progress [9, 30]. The calculations [9], in particular, manifest that, if symmetries of the 4×4 mass matrices required by the theory are respected in all orders in loop corrections, the observed masses and mixing matrices of the lower three families, when included into 4×4 lead to mass matrices quite close (within a factor of 3) to the democratic matrices, equivalently for quarks and leptons.

Appendix: The technique for representing spinors [1, 3, 28, 29]

The technique [1, 3, 28, 29] can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups with the infinitesimal generators of the groups S^{ab} and \tilde{S}^{ab} , as well as transformation properties of the states under any Clifford algebra object γ^a and $\tilde{\gamma}^a$, $\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}$, $\{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}$, $\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$, for any d , even or odd.

Since the Clifford algebra objects $S^{ab} = (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)$ and $\tilde{S}^{ab} = (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$ close the algebra of the Lorentz group, while $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$, S^{ab} and \tilde{S}^{ab} form the equivalent representations to each other. If S^{ab} are used to determine spinor representations in d dimensional space, and after the break of symmetries, the spin and the charges in $d = (1+3)$, can \tilde{S}^{ab} be used to describe families of spinors.

To make the technique simple the graphic presentation of nilpotents and projectors was introduced [29]. For even d we have

$$\begin{pmatrix} ab \\ k \end{pmatrix} = \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b), \quad \begin{pmatrix} ab \\ [k] \end{pmatrix} = \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b), \quad (\text{A.1})$$

with the properties $k^2 = \eta^{aa}\eta^{bb}$ and

$$S^{ab} \begin{pmatrix} ab \\ k \end{pmatrix} = \frac{1}{2}k \begin{pmatrix} ab \\ k \end{pmatrix}, \quad S^{ab} \begin{pmatrix} ab \\ [k] \end{pmatrix} = \frac{1}{2}k \begin{pmatrix} ab \\ [k] \end{pmatrix}, \quad \tilde{S}^{ab} \begin{pmatrix} ab \\ k \end{pmatrix} = \frac{1}{2}k \begin{pmatrix} ab \\ k \end{pmatrix}, \quad \tilde{S}^{ab} \begin{pmatrix} ab \\ [k] \end{pmatrix} = -\frac{1}{2}k \begin{pmatrix} ab \\ [k] \end{pmatrix}. \quad (\text{A.2})$$

One recognizes that γ^a transform $\begin{pmatrix} ab \\ k \end{pmatrix}$ into $[-k]$, never to $\begin{pmatrix} ab \\ [k] \end{pmatrix}$, while $\tilde{\gamma}^a$ transform $\begin{pmatrix} ab \\ k \end{pmatrix}$ into $\begin{pmatrix} ab \\ [k] \end{pmatrix}$, never to $[-k]$

$$\begin{aligned} \gamma^a \begin{pmatrix} ab \\ k \end{pmatrix} &= \eta^{aa} \begin{pmatrix} ab \\ -k \end{pmatrix}, \quad \gamma^b \begin{pmatrix} ab \\ k \end{pmatrix} = -ik \begin{pmatrix} ab \\ -k \end{pmatrix}, \quad \gamma^a \begin{pmatrix} ab \\ [k] \end{pmatrix} = \begin{pmatrix} ab \\ -k \end{pmatrix}, \quad \gamma^b \begin{pmatrix} ab \\ [k] \end{pmatrix} = -ik\eta^{aa} \begin{pmatrix} ab \\ -k \end{pmatrix}, \\ \tilde{\gamma}^a \begin{pmatrix} ab \\ k \end{pmatrix} &= -i\eta^{aa} \begin{pmatrix} ab \\ [k] \end{pmatrix}, \quad \tilde{\gamma}^b \begin{pmatrix} ab \\ k \end{pmatrix} = -k \begin{pmatrix} ab \\ [k] \end{pmatrix}, \quad \tilde{\gamma}^a \begin{pmatrix} ab \\ [k] \end{pmatrix} = i \begin{pmatrix} ab \\ k \end{pmatrix}, \quad \tilde{\gamma}^b \begin{pmatrix} ab \\ [k] \end{pmatrix} = -k\eta^{aa} \begin{pmatrix} ab \\ k \end{pmatrix}. \end{aligned} \quad (\text{A.3})$$

Let us add some useful relations

$$\begin{aligned} \begin{pmatrix} ab \\ k \end{pmatrix} \begin{pmatrix} ab \\ k \end{pmatrix} &= 0, \quad \begin{pmatrix} ab \\ k \end{pmatrix} \begin{pmatrix} ab \\ -k \end{pmatrix} = \eta^{aa} \begin{pmatrix} ab \\ [k] \end{pmatrix}, \quad \begin{pmatrix} ab \\ [k] \end{pmatrix} \begin{pmatrix} ab \\ [k] \end{pmatrix} = \begin{pmatrix} ab \\ [k] \end{pmatrix}, \quad \begin{pmatrix} ab \\ [k] \end{pmatrix} \begin{pmatrix} ab \\ -k \end{pmatrix} = 0, \\ \begin{pmatrix} ab \\ k \end{pmatrix} \begin{pmatrix} ab \\ [k] \end{pmatrix} &= 0, \quad \begin{pmatrix} ab \\ [k] \end{pmatrix} \begin{pmatrix} ab \\ k \end{pmatrix} = \begin{pmatrix} ab \\ k \end{pmatrix}, \quad \begin{pmatrix} ab \\ -k \end{pmatrix} \begin{pmatrix} ab \\ [k] \end{pmatrix} = \begin{pmatrix} ab \\ -k \end{pmatrix}, \quad \begin{pmatrix} ab \\ [k] \end{pmatrix} \begin{pmatrix} ab \\ -k \end{pmatrix} = 0. \end{aligned} \quad (\text{A.4})$$

Defining

$$\begin{pmatrix} ab \\ \tilde{i} \end{pmatrix} = \frac{1}{2}(\tilde{\gamma}^a \mp \tilde{\gamma}^b), \quad (\pm 1) = \frac{1}{2}(\tilde{\gamma}^a \pm i\tilde{\gamma}^b), \quad (\text{A.5})$$

it follows

$$\begin{pmatrix} ab \\ \tilde{k} \end{pmatrix} \begin{pmatrix} ab \\ k \end{pmatrix} = 0, \quad \begin{pmatrix} ab \\ -k \end{pmatrix} \begin{pmatrix} ab \\ k \end{pmatrix} = -i\eta^{aa} \begin{pmatrix} ab \\ [k] \end{pmatrix}, \quad \begin{pmatrix} ab \\ \tilde{k} \end{pmatrix} \begin{pmatrix} ab \\ [k] \end{pmatrix} = i \begin{pmatrix} ab \\ k \end{pmatrix}, \quad \begin{pmatrix} ab \\ \tilde{k} \end{pmatrix} \begin{pmatrix} ab \\ -k \end{pmatrix} = 0. \quad (\text{A.6})$$

We define the vacuum $|\psi_0\rangle$ so that $\langle \begin{pmatrix} ab \\ k \end{pmatrix}^\dagger \begin{pmatrix} ab \\ k \end{pmatrix} \rangle = 1$ and $\langle \begin{pmatrix} ab \\ [k] \end{pmatrix}^\dagger \begin{pmatrix} ab \\ [k] \end{pmatrix} \rangle = 1$.

Making a choice of the Cartan subalgebra set of the algebra S^{ab} and \tilde{S}^{ab} ($S^{03}, S^{12}, S^{56}, S^{78}, \dots$) and ($\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \dots$) an eigen state of all the members of the Cartan subalgebra, representing a weak chargeless u_R -quark with spin up, hyper charge $(2/3)$ and colour $(1/2, 1/(2\sqrt{3}))$, for example, can be written as $(+i)(+) \mid \begin{smallmatrix} 03 & 12 & 56 & 78 \\ (+) & (+) & (+) & (+) \end{smallmatrix} \mid \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (-) & (-) & (-) & (-) & (-) \end{smallmatrix} \mid \psi\rangle = \frac{1}{2^7}(\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2) |(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8) |(\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi\rangle$. This state is an eigen state of all S^{ab} and \tilde{S}^{ab} which are members of the Cartan subalgebra. The definition of the charges can be found in the ref. [2, 3].

i		$ \psi_i\rangle$	$\Gamma^{(1,3)}$	S^{12}	τ^{13}	Y	Q
		Octet of quarks					
1	u_R^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9\ 10 & 11\ 12 & 13\ 14 \\ (+i) & (+) & & (+) & (+) & & (+) & [-] & [-] \end{smallmatrix}$	1	$\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{2}{3}$
2	u_R^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9\ 10 & 11\ 12 & 13\ 14 \\ [-i] & [-] & & (+) & (+) & & (+) & [-] & [-] \end{smallmatrix}$	1	$-\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{2}{3}$
3	d_R^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9\ 10 & 11\ 12 & 13\ 14 \\ (+i) & (+) & & [-] & [-] & & (+) & [-] & [-] \end{smallmatrix}$	1	$\frac{1}{2}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
4	d_R^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9\ 10 & 11\ 12 & 13\ 14 \\ [-i] & [-] & & [-] & [-] & & (+) & [-] & [-] \end{smallmatrix}$	1	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
5	d_L^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9\ 10 & 11\ 12 & 13\ 14 \\ [-i] & (+) & & [-] & (+) & & (+) & [-] & [-] \end{smallmatrix}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
6	d_L^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9\ 10 & 11\ 12 & 13\ 14 \\ (+i) & [-] & & [-] & (+) & & (+) & [-] & [-] \end{smallmatrix}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
7	u_L^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9\ 10 & 11\ 12 & 13\ 14 \\ [-i] & (+) & & (+) & [-] & & (+) & [-] & [-] \end{smallmatrix}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
8	u_L^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 & 9\ 10 & 11\ 12 & 13\ 14 \\ (+i) & [-] & & (+) & [-] & & (+) & [-] & [-] \end{smallmatrix}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$

TABLE II: The 8-plet of quarks - the members of $SO(7,1)$ subgroup of the group $SO(13,1)$, belonging to one Weyl spinor representation of $SO(13,1)$ is presented in the technique [29]. It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular colour $(1/2, 1/(2\sqrt{3}))$. Here $\Gamma^{(1,3)}$ defines the handedness in $(1+3)$ space, S^{12} defines the ordinary spin, τ^{13} defines the third component of the weak charge, Y is the hyper charge, $Q = Y + \tau^{13}$ is the electromagnetic charge. The vacuum state $|\psi_0\rangle$, on which the nilpotents and projectors operate, is not shown. The basis is the massless one. One easily sees that $\gamma^0 \begin{smallmatrix} 78 \\ (-) \end{smallmatrix}$ transforms the first line (u_R^{c1}) into the seventh one (u_L^{c1}), and $\gamma^0 \begin{smallmatrix} 78 \\ (+) \end{smallmatrix}$ transforms the third line (d_R^{c1}) into the fifth one (d_L^{c1}).

In Table II the eightplet of quarks of a particular colour charge ($\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3})$) and the $U(1)_{II}$ charge ($\tau^4 = 1/6$) is presented in our technique [28, 29], as products of nilpotents and projectors. The operators \tilde{S}^{ab} generate families from the starting u_R quark, transforming u_R quark from Table (I) to the u_R of another family, keeping all the properties with respect to S^{ab} unchanged. The eight families of the first member of the eightplet of quarks from Table II, for example, that is of the right handed u_R^{c1} -quark with spin $\frac{1}{2}$, are presented in the left column of Table III. The eight-plet of the corresponding right handed neutrinos with spin up is presented in the right column of the same table. All the other members of any of the eight families of quarks or leptons follow from any member of a

I_R	u_R^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 \\ (+i) & (+) & (+) & (+) \end{smallmatrix} \parallel \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (-) & (-) & (-) & (-) & (-) \end{smallmatrix}$	ν_R	$\begin{smallmatrix} 03 & 12 & 56 & 78 \\ [+i] & (+) & (+) & (+) \end{smallmatrix} \parallel \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (+) & (+) & (+) & (+) & (+) \end{smallmatrix}$
II_R	u_R^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 \\ [+i] & (+) & (+) & (+) \end{smallmatrix} \parallel \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (-) & (-) & (-) & (-) & (-) \end{smallmatrix}$	ν_R	$\begin{smallmatrix} 03 & 12 & 56 & 78 \\ (+i) & (+) & (+) & (+) \end{smallmatrix} \parallel \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (+) & (+) & (+) & (+) & (+) \end{smallmatrix}$
III_R	u_R^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 \\ (+i) & (+) & (+) & (+) \end{smallmatrix} \parallel \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (-) & (-) & (-) & (-) & (-) \end{smallmatrix}$	ν_R	$\begin{smallmatrix} 03 & 12 & 56 & 78 \\ (+i) & (+) & (+) & (+) \end{smallmatrix} \parallel \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (+) & (+) & (+) & (+) & (+) \end{smallmatrix}$
IV_R	u_R^{c1}	$\begin{smallmatrix} 03 & 12 & 56 & 78 \\ [+i] & (+) & (+) & (+) \end{smallmatrix} \parallel \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (-) & (-) & (-) & (-) & (-) \end{smallmatrix}$	ν_R	$\begin{smallmatrix} 03 & 12 & 56 & 78 \\ [+i] & (+) & (+) & (+) \end{smallmatrix} \parallel \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ (+) & (+) & (+) & (+) & (+) & (+) \end{smallmatrix}$

TABLE III: Four families of the right handed u_R^{c1} quark with spin $\frac{1}{2}$, the colour charge ($c^1 = (\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3}))$), and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. All the families follow from the starting one by the application of the operators \tilde{S}^{ab} , $a, b \in \{0, 1, 2, \dots, 8\}$. The generators S^{ab} , $a, b \in \{0, 1, 2, \dots, 8\}$ transform equivalently the right handed neutrino ν_R of spin $\frac{1}{2}$ to all the colourless members of the same family.

particular family by the application of the operators S^{ab} on this particular member.

Let us add below some useful relations [2]

$$\begin{aligned}
N_+^\pm &= N_+^1 \pm i N_+^2 = -(\mp i)(\pm), \quad N_-^\pm = N_-^1 \pm i N_-^2 = (\pm i)(\pm), \\
\tilde{N}_+^\pm &= -(\mp i)(\pm), \quad \tilde{N}_-^\pm = (\pm i)(\pm), \\
\tau^{1\pm} &= (\mp)(\pm)(\mp), \quad \tau^{2\mp} = (\mp)(\mp)(\mp), \\
\tilde{\tau}^{1\pm} &= (\mp)(\pm)(\mp), \quad \tilde{\tau}^{2\mp} = (\mp)(\mp)(\mp), \\
\tau^{Ai} &= \sum_{ab} C_{ab}^{Ai} S^{ab}, \quad \tilde{\tau}^{Ai} = \sum_{ab} C_{ab}^{Ai} \tilde{S}^{ab}.
\end{aligned} \tag{A.7}$$

Appendix: Scalars contributing to the *break II*

In section II scalar fields Φ^{IAi} , contributing to masses of the lower four families (among which are the three so far observed families), are discussed. Here we discuss properties of scalar fields Φ^{IIAi} , contributing to masses of the upper four families.

Let Φ^{IIAi} stay for all the scalar fields contributing to massess of the upper four families and to the masses of the gauge vector bosons \vec{A}_m^2 , $m = 0, 1, 2, 3$

$$\begin{aligned}
\Phi^{IIAi} &\equiv \Phi_{\mp}^{IIAi}, \quad \Phi_{\mp}^{IIAi} = (\vec{A}_{\mp}^2, \vec{A}_{\mp}^{\tilde{N}_R}), \\
\Phi_{\mp}^{Ai} &= (\Phi_7^{Ai} \pm i\Phi_8^{Ai}), \quad A_{II} = \{\tilde{2}, \tilde{N}_R\}.
\end{aligned} \tag{A.1}$$

We choose a renormalizable effective potential $V(\Phi^{II,Ai})$ for the (assumed to be) real scalar fields $\Phi^{II,Ai}$ (Eq. (4)), which couple among themselves

$$V(\Phi^{II,Ai}) = \sum_{A,i} \left\{ -\frac{1}{2} (m_{Ai}^{II})^2 (\Phi^{II,Ai})^2 + \frac{1}{4} \sum_{B,j} \lambda^{II,Ai,Bj} (\Phi^{II,Ai})^2 (\Phi^{II,Bj})^2 \right\}, \quad (\text{A.2})$$

with $\lambda^{Ai,Bj} = \lambda^{Bj,Ai}$.

Scalar fields couple to the gauge bosons at the *break II* according to the Lagrange function \mathcal{L}_{sI}

$$\begin{aligned} \mathcal{L}_{sII} &= \sum_{A,i} (p_{0m} \Phi^{II,Ai})^\dagger (p_0^m \Phi^{II,Ai}) - V(\Phi^{II,Ai}), \\ p_{0m} &= p_m - \{g^4 \tau^4 A_m^4 + g^2 \vec{\tau}^2 \vec{A}_m^2\}. \end{aligned} \quad (\text{A.3})$$

The vacuum state can be expressed before the *break II* (due to our technique [2, 28, 29]) as an identity with respect to $SO(4)$ (on which products of nilpotents and projectors, see Table II of appendix , representing fermion states apply). To this vacuum a new kind of terms, manifesting as a doublet with respect to the weak charge and as triplets with respect to the charges of \tilde{S}^{ab} origin, must be added

$$\begin{aligned} (-) \ominus_{II} &= (-) T s_{\tilde{N}_R} |([-](+)) T d_{(-)\vec{\tau}^2} || \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ [-] & [-] & [-] & [-] & [-] & [-] \end{smallmatrix}, \\ (+) \oplus_{II} &= (+) T s_{\tilde{N}_R} |([+](-)) T d_{(+)\vec{\tau}^2} || \begin{smallmatrix} 9 & 10 & 11 & 12 & 13 & 14 \\ [+] & [+] & [+] & [+] & [+] & [+] \end{smallmatrix}. \end{aligned} \quad (\text{A.4})$$

Here $T s_{\tilde{N}_R}$ denotes a triplet with respect to the operators \vec{N}_R and a singlet with respect to \vec{N}_R , while $\begin{smallmatrix} 56 & 78 \\ [+] & [-] \end{smallmatrix} T d_{(\mp)\vec{\tau}^2}$ are the two triplets with respect to $\vec{\tau}^2$ which are doublets with respect to $\vec{\tau}^2$. It is not difficult, using our technique, to define such a vacuum as products of two spinor representations. One can check that the products of projectors and nilpotents defining the members of one family and correspondingly of all the families, when applied on these two additional contributions to the vacuum, gives zero. One finds

$$\begin{aligned} \tau^{2+} \tau^{2-} \begin{smallmatrix} 78 \\ (+) \oplus_{II} \end{smallmatrix} &= \begin{smallmatrix} 78 \\ (+) \oplus_{II} \end{smallmatrix}, & \tau^{2-} \tau^{2+} \begin{smallmatrix} 78 \\ (-) \ominus_{II} \end{smallmatrix} &= \begin{smallmatrix} 78 \\ (-) \ominus_{II} \end{smallmatrix}, \\ Y \begin{smallmatrix} 78 \\ (+) \oplus_{II} \end{smallmatrix} &= 0 = Y \begin{smallmatrix} 78 \\ (-) \ominus_{II} \end{smallmatrix}, & Q \begin{smallmatrix} 78 \\ (+) \oplus_{II} \end{smallmatrix} &= 0 = Q \begin{smallmatrix} 78 \\ (-) \ominus_{II} \end{smallmatrix}, \\ Y' \begin{smallmatrix} 78 \\ (+) \oplus_{II} \end{smallmatrix} &= \frac{1}{2 \cos^2 \theta_2}, & Y' \begin{smallmatrix} 78 \\ (-) \ominus_{II} \end{smallmatrix} &= -\frac{1}{2 \cos^2 \theta_2}, \\ (\tau^{1+}, \tau^{1-}, \tau^{13}) \begin{smallmatrix} 78 \\ (+) \oplus_{II} \end{smallmatrix} &= 0 = (\tau^{1+}, \tau^{1-}, \tau^{13}) \begin{smallmatrix} 78 \\ (-) \ominus_{II} \end{smallmatrix}. \end{aligned} \quad (\text{A.5})$$

We see that scalar fields $\phi_{\mp}^{II,Ai}$ (Eq.4) bring masses to the gauge vector bosons $A_m^{2\pm}$ and $A_m^{Y'} = \cos \theta_2 A_m^{23} - \sin \theta_2 A_m^4$, while $A_m^Y = \sin \theta_2 A_m^{23} + \cos \theta_2 A_m^4$ stays massless, provided that $\frac{g^2}{g^4} \tan \theta_2 = 1$. Also the weak gauge vectors \vec{A}_m^1 stay massless.

At each break the vacuum changes (Eq. (A.4)). At the *break II* it changes to $(I + \overset{78}{(-)} \ominus_{II} + \overset{78}{(+)} \oplus_{II})$. Q and Y of this vacuum are 0, and due to that fact that the expectation values of $\vec{\tau}^2$ and \vec{N}_R are not influenced by $\vec{\tau}^2$ or Y' , one easily finds for the mass term of the $SU(2)_{II}$ and $U(1)_{II}$ vector gauge fields (see Eq. (A.5) and the text below this equation) after the *break II*

$$\left(\frac{1}{2}\right)^2 (g^2)^2 v_{II}^2 \left(\frac{1}{(\cos \theta_2)^2} A_m^{Y'} A^{Y' m} + 2 A_m^{2+} A^{2- m} \right). \quad (\text{A.6})$$

To the vacuum expectation value v_{II} all the scalar fields $\Phi_{\mp}^{II Ai}$ contribute.

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[32] Several colleagues, first of all is H. Bech Nielsen, and students are participating in this project.

[33] More about the two kinds of the Clifford algebra objects can be found in the refs. [1–3, 29].

The appendix gives a short overview of the technique.

[34] (We have: $Y = \tau^4 + \tau^{23}$, $Y' = -\tan^2 \theta_2 \tau^4 + \tau^{23}$, $\vec{\tau}^2 = \frac{1}{2} (S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78})$, $\vec{\tau}^4 = -\frac{1}{3} (S^{910} + S^{1112} + S^{1314})$, $\vec{\tau}^2 = \frac{1}{2} (\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78})$, $\vec{N}_R = \frac{1}{2} (\tilde{S}^{23} - i\tilde{S}^{01}, \tilde{S}^{31} - i\tilde{S}^{02}, \tilde{S}^{12} - i\tilde{S}^{03})$.

[35] (We have: $Q = Y + \tau^{13}$, $Q' = -\tan^2 \theta_1 Y + \tau^{13}$, $\vec{\tau}^1 = \frac{1}{2} (S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78})$, $\vec{\tau}^1 = \frac{1}{2} (\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78})$, $\vec{N}_L = \frac{1}{2} (\tilde{S}^{23} + i\tilde{S}^{01}, \tilde{S}^{31} + i\tilde{S}^{02}, \tilde{S}^{12} + i\tilde{S}^{03})$.

[36] Let me here refer to the simple case of subsect. III A by paying attention to the reader that in Table I the two vacuum expectation values of each of the two scalar triplets, $(\tilde{a}^{\tilde{N}_L \mp}, \tilde{a}^{\tilde{N}_L 3})$ and $(\tilde{a}^{1 \mp}, \tilde{a}^{13})$, are expected to have the property $(\tilde{a}^{\tilde{N}_L +} \approx \tilde{a}^{\tilde{N}_L -})$ and $(\tilde{a}^{1+} \approx \tilde{a}^{1-})$, respectively, or at least very close to this. Then superposition of the scalar fields, to which different families couple, might differ a lot.